OBJECTIVE MATHEMATICS

Volume 2

Descriptive Test Series

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CHAPTER-10 : DIFFERENTIATIAL EQUATIONS

UNIT TEST-1

1. If the solution curve of the differential equation $(y - 2\log_e x)dx + (x \log_e x^2)dy = 0$, $x > 1$ passes through the points $\left(e, \frac{4}{3}\right)$ and (e^4, α) then α is equal to ____.

2. Let the solution curve $x = x(y)$, $0 < y < \frac{\pi}{2}$, of the differential equation $(\log_e(\cos y))^2 \cos y\, dx - (1 + 3x \log_e (\cos y)) \sin y\, dy = 0$ satisfy $x\left(\frac{\pi}{3}\right) = \frac{1}{2\log_e 2}$. If $x\left(\frac{\pi}{6}\right) = \frac{1}{\log_e m - \log_e n}$, where m and n are co-prime, then mn is equal to

3. Let $y = y(x)$ be a solution of the differential equation $(x\cos x)dy + (xy\sin x + y\cos x - 1)dx = 0, 0 < x < \frac{\pi}{2}$. If $\frac{\pi}{3} y\left(\frac{\pi}{3}\right) = \sqrt{3}$ then $\left|\frac{\pi}{6} y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right|$ is equal to _______.

Hints and Solutions

1. (03.00)

$\because$ $(y - 2\ln x)dx + (2x\ln x)dy = 0.$

$$2x\ln x\frac{dy}{dx} + y = 2\ln x$$

$$\frac{dy}{dx} + \frac{y}{2x\ln x} = \frac{1}{x}$$

$\therefore$ $\text{I.F.} = e^{\int \frac{1dx}{2x\ln x}} = d\sqrt{\ln x}$

$\therefore$ Solution of the equation is:

$$y \cdot \sqrt{\ln x} = \int \frac{\sqrt{\ln x}}{x} = dx$$

$\therefore$ $y \cdot \sqrt{\ln x} = \frac{2}{3}(\ln x)^{\frac{3}{2}} + C$ ...(i)

$\therefore$ eq. (i) passes through point $\left(e, \frac{2}{3}\right)$.

$\therefore$ $\left(c = \frac{2}{3}\right)$

$\therefore$ $y\sqrt{\ln x} = \frac{2}{3}(\ln x)^{\frac{3}{2}} + \frac{2}{3}$ ...(ii)

This equation passes through point (e^4, α)

$\therefore$ $\alpha = 3.$

2. (12) Given equation can be written as,

$$\frac{dx}{dy} + \left(\frac{-3\tan y}{\ln\cos y}\right)x = \frac{\sin y}{(\ln\cos y)^2 \cdot \cos y}$$

Integrating Factor (I.F.) $= e^{3\int \frac{-\tan y}{\ln\cos y}dy} = (\ln\cos y)^3$

Solution of differential equation

$$x \cdot (\ln\cos y)^3 = \int \frac{\sin y}{\cos y}(\ln\cos y)dy$$

$$\Rightarrow x(\ln\cos y)^3 = \frac{-(\ln\cos y)^2}{2} + c$$

$$x\left(\frac{\pi}{3}\right) = \frac{1}{2\ln 2}$$

$\Rightarrow c = 0$

Here, $x = \frac{-1}{2\ln\cos y}$

$$x\left(\frac{\pi}{6}\right) = -\frac{1}{2\ln\frac{\sqrt{3}}{2}} = \frac{1}{\ln 4 - \ln 3}$$

$$m = 4, n = 3$$

$$\Rightarrow \quad m.n. = 12$$

3. (02.00) $\because (x \cos x)dy + (xy \sin x + y \cos x - 1)dx = 0$

$$\therefore \quad (x\cos x)\frac{dy}{dx} + y(x\sin x + y\cos x) = 1$$

$$\Rightarrow \frac{dy}{dx} + y\left(\frac{x\sin x + \cos x}{x\cos x}\right) = \frac{1}{x\cos x}$$

$$\therefore \quad \text{Integrating factor} = e^{\int\left(\tan x + \frac{1}{x}\right)dx}$$

$$= x \sec x$$

$$\therefore \quad y \cdot x\sec x = \int \frac{x\sec x}{x\cos x}dx$$

$$\therefore \quad xy \sec x = \tan x + c$$

$$\because \quad \frac{\pi}{3}y\left(\frac{\pi}{3}\right)\sec\frac{\pi}{3} = \tan\frac{\pi}{3} + c.$$

$$\therefore \quad c = \sqrt{3}$$

$$\therefore \quad xy \sec x = \tan x + \sqrt{3}$$

$$\therefore \quad y(x) = \frac{2\sin\left(x + \frac{\pi}{3}\right)}{x}$$

$$\therefore \quad xy''(x) + 2y'(x) = -2\sin\left(x + \frac{\pi}{3}\right)$$

Thus $\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right) = -2$

Hence $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right| = 2$